the angular division N up to n. Some of the results show the direct n^2 convergence characteristics with the number of divisions n, if $N_\phi = N_\theta = n$.

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Transient Natural Convection in a Cylindrical Annulus

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Nomenclature

g = acceleration caused by gravity, 981 cm/s^2
L_{ref} = characteristic length, $r_o - r_i$, cm
P = nondimensional pressure, $p/\rho U_{ref}^2$
Pr = Prandtl number
p = pressure, dyne/cm^2
R = nondimensional radius, r/L_{ref}
Ra = Rayleigh number, Eq. (5)
r = radius, cm
T = temperature of fluid, K
T_h = temperature of walls
T_{ref} = $T_h - T_{in}$
t = time, s
U = nondimensional radial velocity, u/U_{ref}
U_{ref} = characteristic velocity, α/L_{ref}
u = velocity along radius, cm/s
V = nondimensional axial velocity, v/U_{ref}
v = velocity along Y axis, cm/s
X = nondimensional radial coordinate, x/L_{ref}
Y = nondimensional axial coordinate, y/L_{ref}
α = thermal diffusivity, cm^2/s
β = expansion coefficient, K^{-1}
θ = nondimensional temperature, $(T - T_{in})(T_h - T_{in})$
ν = kinematic viscosity, cm^2/s
ρ = density, g/cm^3
τ = nondimensional time, $t/(L_{ref}^2/\alpha)$

Subscripts
i = inner wall
in = initial condition
o = outer wall

Introduction

NATURAL convective flows of fluids in cylindrical annuli are important in many engineering applications. The flow currents strongly influence the temperature distribution in the fluid and have been studied extensively for alternate hot and cold wall boundaries.[1–3] However, the transient heating of a cylindrical annuli, when the fluid is initially at a temperature lower than the walls, has not been investigated. The results of the transient prediction for a vertically held cylindrical annulus with isothermal inner and outer walls are discussed in this Note. An aspect ratio of unity is taken for the annulus and different values of radius ratios are considered.

Computations

The unsteady governing equations for continuity, momentum, and energy are solved in the axisymmetric coordinate system for U, V, θ, and P. The radial and axial velocities are nondimensionalized with U_{ref} specified as the ratio of α and L_{ref}. This reference length is specified as the radial width of the annulus ($L_{ref} = r_o - r_i$). The temperature θ is expressed as a fraction of the temperature increase of the fluid form its initial value and from T_{ref}. T_{ref} is the difference in temperature between the hot walls T_h and the initial temperature of the fluid T_{in}. The pressure is nondimensionalized by ρU_{ref}^2, where ρ is the density of the fluid. The radial and axial coordinates r and y are nondimensionalized using the reference length L_{ref}. τ is the ratio of the dimensional time t in seconds and the characteristic time L_{ref}^2/α.

The momentum and energy equations in the preceding nondimensional coordinate system considering laminar flow are as follows:

$$\frac{\partial U}{\partial \tau} + \frac{U \partial U}{\partial R} + \frac{V \partial U}{\partial Y} = -\frac{\partial P}{\partial R} + Pr\nabla^2 U \tag{1}$$

$$\frac{\partial V}{\partial \tau} + \frac{U \partial V}{\partial R} + \frac{V \partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr\nabla^2 V + RaPr\theta \tag{2}$$

$$\frac{\partial \theta}{\partial \tau} + \frac{U \partial \theta}{\partial R} + \frac{V \partial \theta}{\partial Y} = \nabla^2 \theta \tag{3}$$

Ra and Pr are the Rayleigh and Prandtl numbers, respectively. The Rayleigh number in Eq. (2) is defined in terms of its initial value at $\tau = 0$ (Ra_{in}) as

$$Ra = Ra_{in}(1 - \theta) \tag{4}$$

where Ra_{in} is the initial Rayleigh number given by

$$Ra_{in} = g\beta(T_h - T_{in})L_{ref}^3/\alpha\nu \tag{5}$$

As the temperature of the fluid increases during the transient heating, the temperature difference between the wall and the fluid ($T_h - T$) decreases and, hence, the buoyancy force reduces. The Rayleigh number in the conservation equation for vertical momentum [Eq. (2)] progressively reduces as time increases. The boundary conditions at the walls are $U = 0$, $V = 0$, and $\theta = 1$. The upper and lower walls are assumed to be insulated.

The penalty formulation of the finite element method given by Taylor and Hughes[3] and Reddy[4] is used to solve the con-

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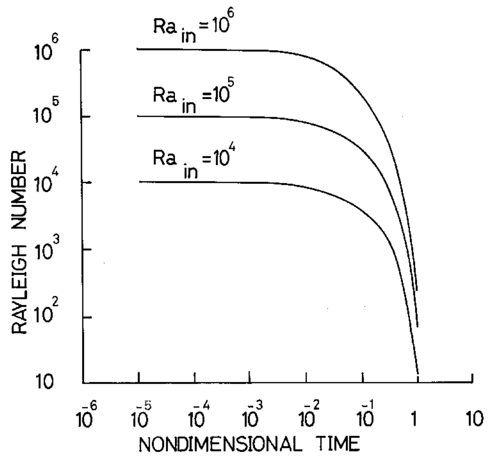

Fig. 1 Decay of Rayleigh numbers caused by heating.

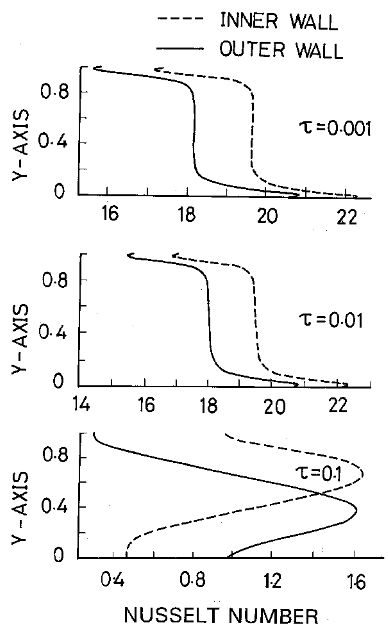

Fig. 2 Variation of Nusselt numbers at different nondimensional times for Ra_{in} = 10^6.

servation equations. Values of velocities and temperatures are considered at the corners of four-sided rectangular elements. The pressure is evaluated at the center of these rectangular elements. Nonuniform elements are fitted in the annulus. The predictions are shown to be independent of number of grids when the Rayleigh number is less than 10^7, provided the number of grids used are greater than 41 × 41. The values of velocities and stream functions, computed by this method, agree well with the available benchmark solutions of Davis[5] for $Ra < 10^6$ when alternate hot and cold wall boundaries are specified. The results are presented for a fluid having a Prandtl number of 0.75. Variations in the Prandtl number did not significantly change the predictions.

Results and Discussions

As the fluid in the annulus gets heated, the value of the Rayleigh number decreases, starting from the initial value (Ra_{in}) at $\tau = 0$. A plot of the variation of the Rayleigh number with time is shown in Fig. 1 for Ra_{in} of 10^4, 10^5, and 10^6. The Rayleigh number is observed to be near the initial value until a nondimensional time of about 10^{-2}. Thereafter it decreases rapidly to insignificantly small values. The flow currents and the heat transfer would therefore persist during the initial phase of the nondimensional times. If an annulus of width of a few

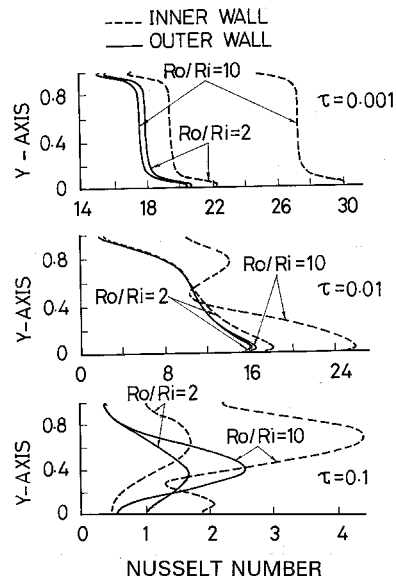

Fig. 3 Variation of Nusselt numbers with radius ratios for Ra_{in} = 10^6.

centimeters is considered, this nondimensional time would be a few seconds if the fluid in the annulus is a gas. It increases to a few minutes when the fluid is a liquid because of the lower thermal diffusivity.

Figure 2 illustrates the variation of the Nusselt number at the inner and outer cylindrical walls at different nondimensional times for an initial Rayleigh number of 10^6. The higher values of flow currents formed initially at the inner wall lead to large values of Nusselt numbers. The velocities at the inner wall get rapidly attenuated when the circulation from the outer wall spreads to the inner wall and arrests the upward velocities at the inner wall. This results in a fall of the Nusselt number at the inner wall as time progresses. The stronger currents formed at the lower corners of the annulus at early nondimensional times give higher values of Nusselt number at these regions. When a nondimensional time of 0.1 is reached, the flow currents have sufficiently weakened, leading to negligibly small values of Nusselt numbers.

Figure 3 shows the variation of Nusselt number distribution along the inner and outer walls caused by changes in the radius ratio of the annulus. An initial Rayleigh number of 10^6 is considered. When the radius ratio of the annulus is increased from 2 to 10, considerable changes in the Nusselt number are observed along the inner wall. This is illustrated in Fig. 3 for a nondimensional time of 0.01. The bottom corners of the inner wall, where the flow currents are sizable, retain relatively higher values of Nusselt numbers, whereas the center region where the vertical velocity is a minimum, gives small values of Nusselt number. The flow currents originating from the outer wall strongly influences the velocities at the inner wall region as the radius ratio of the annulus increases. Larger variations in Nusselt numbers are therefore observed along the inner wall for the higher radius ratios of the annulus. The Nusselt numbers reduce to very small values by the nondimensional time of 0.1 because of the weakening of the buoyancy currents from the rapid drop of the Rayleigh number.

Conclusions

Transient natural convection in a vertically held cylindrical annulus is predicted when the inner and outer walls are kept at the same isothermal condition, which is higher than the initial temperature of the fluid. The Rayleigh number is shown to rapidly decay to negligibly small values for a nondimensional time exceeding 10^{-2}. Initially, the Nusselt number at the inner wall is higher than at the outer walls. The Nusselt number varies significantly along the inner wall, particularly at the larger radius ratio of the annulus. Negligibly small values of Nusselt number are obtained at the inner and outer walls for nondimensional times exceeding 0.1.

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